13. MARKEYEV A.P. and MOSHCHUK N.K., Qualitative analysis of the motion of a heavy rigid body on a smooth horizontal plane. Prikl. Mat. Mekh., 47, 1, 1983.
14. KOZLOV V.V. and KOLESNIKOV N.N., On the theorems of dynamics. Prikl. Mat. Mekh., 42. 1, 1978.
15. ARNOL'D V.I., Mathematical Methods of Classical Mechanics, Moscow, Nauka, 1979.
16. MOSHCHUK N.K., Reduction of the equations of motion of certain non-holonomic Chaplygin systems to the form of Lagrange and Hamilton equations. Prikl. Mat. Mekh., 51, 2, 1987.
17. ARNOL'D V.I., A proof of A.N. Kolmogorov's theorm on the conservation of conditionally-periodic motions under small variations of the Hamiltonian. Uspekhi Mat. Nauk, 18, 5, 1963,
18. MOZER YU., A rapidly converging method of iterations and non-linear differential equations. Uspekhi Mat. Nauk, 23, 4, 1968.
19. MOZER YU., On expansion of conditionally-periodic motions in converging power series. Uspekhi Mat. Nauk 24, 2, 1969.
20. SEVRYUK M.B., Reversible Systems, Berlin, Springer, 1986.

Translated by D.L.

PMM U.S.S.R.,Vol.52,No.2,pp.165-170,1988 0021-8928/88 $10.00+0.00
Printed in Great Britain

DIFFUSION SPREADING OF LOCALIZED HYDRODYNAMIC DISTURBANCES UNDER THE ACTION OF RANDOM FORCES*

V.A. GORODTSOV

The effect of a time-dependent random force on fluid flow may be found by changing to a non-inertial coordinate system. It is shown that, under the action of a Gaussian random force, initially localized disturbances undergo spreading of a diffusion type. Explicit analytic solutions are given for the interior wave soliton under the action of a random force. It is shown that, in the presence of a soliton, the growth of velocity pulsing may either increase or moderate.

1. The evolution of a wide class of one-dimensional disturbances of the velocity field of the flow $u(x, t)$ in hydrodynamics is described by the general non-linear equation /1/

$$u_t + uu_x + \int\limits_{-\infty}^{\infty} dy F(y-x) u_{yy}(y, t) = f \tag{1.1}$$

When there are no external force $(f = 0)$ the Cauchy problem for the homogeneous equation can sometimes be solved by means of reduction to a linear problem, and as a result of the balance of non-linearity, dispersion, and dissipation, the existence of selfpreserving non-linear fields (solitons and shock waves) is possible. In particular, when $F(x) = -\mu\delta(x)$, we obtain Burgers' equation, which, under the Hopf-Cole replacement, reduces to the linear equation of diffusion. For

$$F(x) \sim -\delta'(x), \quad P\frac{1}{x}, \quad P\left(\operatorname{ctg}\frac{\pi x}{2h} - \operatorname{sgn} x\right)$$

the equations are respectively, completely integrable Korteweg- de Vries equations, Benjamin-Ono equations, and the equations of the interior waves in a basin of finite depth (the symbol P indicates that the singular integrals are to be taken in the sense of the principal value). The reducibility to a linear problem in these cases is also well-known /2/.

With regard to the non-uniform Eq.(1.1), by using the equivalence of the action of the spatially homogeneous force $f(t)$ and of a suitable acceleration of the coordinate system, the soltuion of (1.1) for $u(x, t)$ can be reduced by the change of variables

**Prikl.Matem.Mekhan.*,52,2,211-217,1988

$$x - x_0(t) = \xi,\ u(x,t) - u_0(t) = v(\xi,t) \qquad (1.2)$$
$$\partial x_0/\partial t = u_0,\ \partial u_0/\partial t = f(t)$$

to the solution of exactly the same uniform equation for $v(\xi, t)$ (the solution of the uniform equation will henceforth be denoted by the letter v to distinguish it from the solution u of the non-uniform equation).

By using the known solutions of the uniform equation and the above change of variables, we can estimate the influence of the external forces. Under the action of the determinate external force $f(t)$ a uniform "background" flow $u_0(t) = \int dtf$ and a variable motion of the "centre of gravity" of the solution of the uniform equation $x_0(t) = \int dtu_0$ are induced. In particular, in the case of an oscillating external force, both the background and the centre of gravity oscillate. Under the action of a random force, fluctuations of both of these arise.

We shall first confine ourselves to the mean flow caused by a Gaussian random force with zero mean value $\langle f\rangle = 0$. In accordance with transformation (1.2), we have

$$\langle u(x,t)\rangle = \langle v(x - x_0(t), t)\rangle = \left\langle \exp\left(-x_0(t)\frac{\partial}{\partial x}\right)\right\rangle v(x,t) = \qquad (1.3)$$
$$\exp(\tau\partial^2/\partial x^2)\, v(x,t), \quad \tau \equiv \langle x_0^2(t)\rangle/2$$

The last important equation is proved by series expansion of the exponential function, followed by term by term averaging, and reverse transformation of the result into an exponential function. Since the random force is Gaussian, all the odd moments vanish, while the even moments reduce to integral powers of the second moment.

The operator relation (1.3) can also be written as a Fourier expansion or integral convolution:

$$\langle u(x,t)\rangle = \frac{1}{2\pi}\int_{-\infty}^{\infty} dkv(k,t)\exp(-k^2\tau + ikx) = \qquad (1.4)$$
$$\int_{-\infty}^{\infty} dyg(y-x,t)\, v(y,t), \quad g(x,t) = (4\pi\tau)^{-1/2}\exp\left(-\frac{x^2}{4\tau}\right)$$

If we now pose the question of the influence of a Gaussian random force on non-linear waves of soliton type (or shock-wave type), which propagate in the absence of forces without a change of shape $V(x - ct)$, then we obtain from (1.3):

$$\langle u(x,t)\rangle = \exp(\tau\partial^2/\partial\eta^2) V(\eta), \quad \eta = x - ct \qquad (1.5)$$

i.e., under the action of random forces these waves must undergo a diffusion type of smearing:

$$\frac{\partial\langle u\rangle}{\partial\tau} = \frac{1}{D(t)}\frac{\partial\langle u\rangle}{\partial t} = \frac{\partial^2\langle u\rangle}{\partial\eta^2}, \quad D(t) = \frac{1}{2}\frac{\partial}{\partial t}\langle x_0^2(t)\rangle \qquad (1.6)$$

At long times, when τ is large and the main contribution to the Fourier expansion is from small k, we obtain from (1.4) the simplified relation

$$\langle u(x,t)\rangle \approx v(k=0,t) g(x,t), \quad t \to \infty \qquad (1.7)$$

which is suitable when the asymptotic stage is reached and the diffusion width $\sim\tau^{1/2}$ becomes much greater than the initial width of the disturbance.

For random forces of the "white noise" type we have

$$\langle f(t_1) f(t_2)\rangle = f_0^2\delta(t_1 - t_2), \quad \tau = {}^1/_2\langle x_0^2(t)\rangle = {}^1/_6 f_0^2 t^3 \qquad (1.8)$$

and at the remote stage of soliton smearing $(t \to \infty)$ its width (in the mean) increases as $\tau^{1/2} \sim t^{3/2}$, whereas its height decreases as $\tau^{-1/2} \sim t^{-3/2}$. The averaged "soliton area" then remains constant in time:

$$\int_{-\infty}^{\infty} dx\langle u(x,t)\rangle = V \quad (k=0) \qquad (1.9)$$

which is the result of the initial Eq.(1.1) with $\langle f\rangle = 0$.

An example of a completely integrable uniform equation of type (1.1) is the interior wave equation /2/

$$v_t + vv_x + \frac{1+h}{2h^2}\int d\xi\left[\operatorname{cth}\frac{\pi(\xi - x)}{2h} - \operatorname{sgn}(\xi - x)\right] v_{\xi\xi}(\xi,t) = 0 \qquad (1.10)$$

which has a solution of the soliton type (with $0 < \varkappa h < \pi$)

$$V(\eta) = \left(1 + \frac{1}{h}\right)\frac{2\varkappa\sin\varkappa h}{\cos\varkappa h + \operatorname{ch}\varkappa\eta} = \qquad (1.11)$$

$$2\left(1+\frac{1}{h}\right)\int_{-\infty}^{\infty} dk\,\frac{\operatorname{sh} kh}{\operatorname{sh} k\pi\varkappa^{-1}}\exp(ik\eta)$$

The smearing of this soliton when random forces come into action (at the instant $t = 0$) is given by a Fourier expansion of type (1.4):

$$\langle u(x,t)\rangle = 2\left(1+\frac{1}{h}\right)\int_{-\infty}^{\infty} dk\,\frac{\operatorname{sh} kh}{\operatorname{sh} k\pi\varkappa^{-1}}\exp(-k^2\tau + ik\eta) \tag{1.12}$$

At long times $(\tau \to \infty)$ the soliton height, given by $\langle u\rangle$ with $\eta = 0$, is $2\varkappa(1 + h)(\pi\tau)^{-1/2}$. Since the soliton area is constant: $V(k = 0) = 4\varkappa(1 + h)$, its width (the ratio of the area to the height) will increase as $2(\pi\tau)^{1/2}$.

In the "shallow water" limit $(h \to 0, \varkappa h \to 0)$ the free soliton of interior waves (1.11) takes the classical form $\varkappa^2/\operatorname{ch}^2(\varkappa\eta/2)$ and satisfies the Korteweg - de Vries equation $v_t + vv_x + {}^1/_3 v_{xxx} = 0$. The behaviour of this soliton in a field of random force is analysed in /3/, where the solution was given in the form (1.12) with k instead of $(1 + 1/h)\operatorname{sh} kh$.

In the "deep water" limit $(h \to \infty, \varkappa \to 0, \varkappa h = \pi - \varkappa/c)$ the free soliton satisfies the Benjamin-Ono equation and has the Lorentz form

$$v_t + vv_x + \frac{1}{\pi}\int_{-\infty}^{\infty} d\xi\,\frac{v_{\xi\xi}(\xi,t)}{\xi - x} = 0, \quad V(\eta) = \frac{4c}{1 + c^2\eta^2} \tag{1.13}$$

Gaussian white noise leads to smearing of the soliton, which is described by (1.12) with $(1 + 1/h)\operatorname{sh} kh/\operatorname{sh} k\pi\varkappa^{-1}$ replaced by $\exp(-|k|/c)$. The result can be written in terms of the error function:

$$\langle u(x,t)\rangle = (\pi/\tau)^{1/2}\exp\zeta^2 \operatorname{erfc}\zeta + \text{c.c.},\quad 2\zeta\tau^{1/2} \equiv i\eta + 1/c \tag{1.14}$$

For the height, width and area of the soliton we have respectively $2(\pi/\tau)^{1/2}$, $2(\pi\tau)^{1/2}$ and 4π.

With regard to the interaction of several solitons, it is well-known /1, 2/ that, after a long time, the solution of the problem is close to the sum of the free solitons in uniform motion. Under the action of random forces, at long times t, the widening of the solitons $(\sim t^{3/2})$ is at a faster rate than their "pick-up" $(\sim t)$, and at the remote stage of evolution all the solitons "merge" into a single weak disturbance /4/.

2. The above analysis can be extended directly to the multidimensional case. For instance, the three-dimensional inhomogeneous Navier-Stokes equations, describing the flow of a homogeneous incompressible fluid (of density $\rho = 1$), under the action of forces $\mathbf{f}(t)$ can be reduced by the change of variables (compare with (1.2))

$$x_i - x_{i0}(t) = \xi_i,\ u_i(\mathbf{x},t) - u_{i0}(t) = v_i(\boldsymbol{\xi},t)$$
$$\partial x_{0i}/\partial t = u_{i0},\ \partial u_{i0}/\partial t = f_i(t)$$

to the same uniform equations for $v_i(\boldsymbol{\xi},t)$.

Hence we find, in just the same way as in the uniform case, that

$$\langle u_i(\mathbf{x},t)\rangle = \langle v_i(\mathbf{x} - \mathbf{x}_0(t), t)\rangle = \exp(\tau\Delta)v_i(\mathbf{x},t) = (2\pi)^{-3}\int d^3k v_i(\mathbf{k},t)\exp(-k^2\tau + i\mathbf{kx}) \tag{2.1}$$

if the random vector force function is assumed to be Gaussian and isotropic. The effect of the random forces on the mean flow is here via a single scalar characteristic τ, in terms of which the correlation $\langle x_{i0}(t)x_{j0}(t)\rangle = 2\tau\delta_{ij}$ is expressed.

From (2.1) as $\tau \to \infty$ we have the simple asymptotic estimate (compare with (1.7))

$$\langle u_i(\mathbf{x},t)\rangle \approx (4\pi\tau)^{-3/2} v_i(k = 0, t)\exp[-\mathbf{x}^2/(4\tau)]$$

In the three-dimensional case also, therefore, the effect of a Gaussian time-dependent random force amounts to diffusion broadening of the initially localized disturbances. Under the action of intense white noise the diffusion smearing of a disturbance which has earlier retained its form, occurs in the same way as in the one-dimensional case $(\sim t^{3/2})$, while the amplitude falls more rapidly as a result of geometric divergence $(\sim t^{-9/2})$. Then, the integral

$$\int d^3x\langle\mathbf{u}(\mathbf{x},t)\rangle = \mathbf{V}(\mathbf{k} = 0)$$

does not vary with time.

3. We will now consider the higher moments of the field of flows caused by a random

force when there is an initial disturbance v. We shall confine ourselves to the one-dimensional model (1.1). By (1.2), the velocity $u(x, t)$ of such flows is the sum $u_0(t) + v(x - x_0(t), t)$ with random functions $u_0(t)$, $x_0(t)$ and the regular **solution** $v(x, t)$ of the uniform equation of type (1.1). Thus evaluation of the moments of the velocity field $u(x, t)$ reduces to finding the correlations of the "background" $u_0(t)$ and the function v of the random variables $x_0(t)$:

$$\left\langle \prod_i u(x_i, t_i) \right\rangle = \left\langle \prod_i u_{0i} \right\rangle + \left\langle u_{01} \prod_{i \neq 1} v(x_i - x_{0i}, t_i) \right\rangle + \ldots + \left\langle \prod_i v(x_i - x_{0i}, t_i) \right\rangle \tag{3.1}$$

Here and below, we use the abbreviated notation $x_0(t_i) \equiv x_{0i}$, $u_0(t_i) \equiv u_{0i}$.

Writing the function $v(x - x_0(t), t)$ as the product of the random shift operator $\exp(-x_0(t)\partial/\partial x)$ and the regular function $v(x, t)$, we find that

$$\left\langle \prod_i v(x_i - x_{0i}, t_i) \right\rangle = \left\langle \prod_i \exp\left(-x_{0i} \frac{\partial}{\partial x_i}\right) \right\rangle \prod_k v(x_k, t_k) = \exp\left(\frac{1}{2} \sum_{i,j} \langle x_{0i} x_{0j} \rangle \frac{\partial^2}{\partial x_i \partial x_j}\right) \prod_k v(x_k, t_k) \tag{3.2}$$

if we use the generalization of (1.3) for an arbitrary Gaussian random field $a(t)$ $(a_i \equiv a(t_i))$ and parameters α_i which may be numbers of operators $\partial/\partial x_i$

$$\left\langle \prod_i \exp(\alpha_i a_i) \right\rangle = \exp\left(\frac{1}{2} \sum_{i,j} \alpha_i \alpha_j \langle a_i a_j \rangle\right) \tag{3.3}$$

For simultaneous correlations (with $t_1 = \ldots = t_n = t$), from (3.2) we have

$$\left\langle \prod_i v(x_i - x_0(t), t) \right\rangle = \exp\left(\tau \frac{\partial^2}{\partial x^2}\right) \prod_i v(x_i, t) \qquad \tau = \frac{1}{2} \langle x_0^2(t) \rangle, \quad x = \frac{1}{n} \sum_i x_i \tag{3.4}$$

so that these correlations satisfy the same diffusion Eq. (1.6) as the mean velocity $\langle u(x, t) \rangle = \langle v(x - x_0(t), t) \rangle$.

For simultaneous correlations at the same point of space $(x_1 = \ldots = x_n = x)$ we have

$$\langle v^n(x - x_0(t), t) \rangle = \exp(\tau \partial^2/\partial x^2) v^n(x, t) = g * v^n \equiv \int dy\, g(x - y, \tau) v^n(y, t), \; g(x, \tau) = \exp(\tau \partial^2/\partial x^2) \delta(x) \tag{3.5}$$

Consequently, the problem of the one-point simultaneous moments $\langle v^n \rangle$ of the field of flows due to a Gaussian random force (switched on at $t = 0$) and an initially stationarily moving disturbance $V(x - ct)$, amounts to solving the simple initial value problem

$$\frac{\partial}{\partial \tau} \langle v^n \rangle = \frac{\partial^2}{\partial \eta^2} \langle v^n \rangle, \quad \langle v^n \rangle|_{\tau=0} = V^n(\eta) \tag{3.6}$$

With $n = 1$ we have the result of Sect.1.

To find the correlation functions of the field $u(x, t)$, we have to find in accordance with (3.1) the correlation also between the background $u_0(t)$ and $v(x - x_0(t), t)$.

For this, we can use the corollary of (3.3):

$$\langle \Pi \rangle = E, \quad \langle a_1 \Pi \rangle = \sum_i \alpha_i \langle a_1 a_i \rangle E$$
$$\langle a_1 a_2 \Pi \rangle = \left(\langle a_1 a_2 \rangle + \sum_{i,j} \alpha_i \alpha_j \langle a_1 a_i \rangle \langle a_2 a_j \rangle \right) E, \ldots$$
$$\Pi \equiv \prod_i \exp(\alpha_i a_i), \quad E \equiv \exp\left\langle \left(\sum_k \alpha_k a_k\right)^2 / 2 \right\rangle \tag{3.7}$$

these relations being obtained by differentiation **of** (3.3) with respect to certain parameters α_i, which are then equated to zero.

Using (3.7), we can obtain e.g.,

$$\langle u_0(t_1) v(x - x_0(t_2), t_2) \rangle = -\langle u_0(t_1) x_0(t_2) \rangle \frac{\partial}{\partial x} \langle u(x, t_2) \rangle \tag{3.8}$$

All in all, the correlation functions of the velocity field $u(x, t)$ are expressed in terms of the second moments of the random functions $u_0(t)$, $x_0(t)$, which are in turn expressible in terms of the correlation function of the Gaussian random force $\langle f(t_1)f(t_2)\rangle$. In the case of a delta correlated force of the type (1.8) we have

$$\langle u_0(t_1)u_0(t_2)\rangle = f_0^2 \min(t_1, t_2) \tag{3.9}$$
$$\langle u_0(t_1)x_0(t_2)\rangle = \frac{1}{2} f_0^2 \begin{cases} 2t_1t_2 - t_1^2, & t_1 < t_2 \\ t_2^2, & t_1 > t_2 \end{cases}$$
$$\langle x_0(t_1)x_0(t_2)\rangle = \frac{1}{6} f_0^2 \begin{cases} 3t_1^2t_2 - t_1^3 & t_1 < t_2 \\ 3t_1t_2^2 - t_2^3 & t_1 > t_2 \end{cases}$$

and in particular, for the simultaneous correlations $\langle u_0^2(t)\rangle = f_0^2 t$, $\langle u_0(t)x_0(t)\rangle = {}^1/_2 f_0^2 t^2$, $2\tau \equiv \langle x_0^2(t)\rangle = {}^1/_3 f_0^2 t^3$.

For the velocity variance $\langle u'^2(x,t)\rangle = \langle u^2\rangle - \langle u\rangle^2$, we can write, in view of the above:

$$\langle u'^2\rangle = \langle u_0^2\rangle - 2\langle u_0x_0\rangle \frac{\partial}{\partial x} g * v + g * v^2 - (g * v)^2 \tag{3.10}$$
$$g * \varphi \equiv \int dy g(x-y,\tau)\varphi(y,t) = \exp\left(\tau \frac{\partial^2}{\partial x^2}\right)\varphi(x,t)$$

It is easy to estimate in the case of short times (from the instant of switching on the random force). Using series expansion in τ, in the case of an initially stationarily moving regular disturbance $V(\eta)$, we find that

$$\langle u'^2\rangle = \langle u_0^2\rangle - 2\langle u_0x_0\rangle \partial V(\eta)/\partial\eta + \langle x_0^2\rangle(\partial V(\eta)/\partial\eta)^2 + o(\tau)$$

Since a shape with one vertex is typical for a soliton, ahead of it (where $\partial V/\partial\eta < 0$) at short times the mean square velocity pulsations will increase more rapidly, and behind it (where $\partial V/\partial\eta > 0$) their increase will slow down due to the presence of the soliton.

To make the estimates for long times $(\tau \to \infty)$ it is better to use series expansions, not of the initial quantities $V(\eta)$, but of their Fourier images $V(k)$ (see (1.4), (1.7)).

Assuming that, for a symmetric soliton solution, $V(k)$ is series-expanded in the neighbourhood of $k=0$ (the expansion is then, by the symmetry, in even powers of k^2), we find that (H_n are Hermite polynomials)

$$\langle u\rangle = g * V = \sum_{m=0}^{\infty} \frac{(-1)^m}{2m!} V^{(2m)} \frac{\partial^{2m} g}{\partial\eta^{2m}} = \tag{3.11}$$
$$g \sum_{m=0}^{\infty} \frac{1}{2m!}\left(-\frac{1}{4\tau}\right)^m V^{(2m)} H_{2m} = \sum_{m=0}^{\infty} \frac{(-1)^m}{2m!} V^{(2m)} \frac{\partial^m g}{\partial\tau^m}$$
$$V^{(2m)} \equiv \frac{\partial^{2m} V(k)}{\partial k^{2m}}\bigg|_{k=0}, \quad \left(\frac{\partial}{\partial\tau} - \frac{\partial^2}{\partial\eta^2}\right) g(\eta,\tau) = 0, \quad H_n = H_n\left(\frac{\eta}{2\tau^{1/2}}\right)$$

By the usual definition of Bernoulli polynomials $B_n(x)$ in terms of the generating function $t\exp(xt)/(\exp t - 1)$, we have

$$\frac{\operatorname{sh}\alpha k}{\operatorname{sh}\beta k} = 2\sum_{m=0}^{\infty} \frac{(2\beta k)^{2m}}{(2m+1)!} B_{2m+1}\left(\frac{\alpha+\beta}{2\beta}\right)$$

which can be used to find the coefficients of the expansion about $k=0$ of the Fourier image of the soliton of internal waves (1.11)

$$V^{(n)} = \delta_{n,2m} \frac{8\pi(1+h)}{2m+1}\left(\frac{2\pi}{\varkappa}\right)^{2m} B_{2m+1}\left(\frac{\pi+\varkappa h}{2\pi}\right)$$

For the soliton there follows in particular from this the Korteweg- de Vries equation (as $h \mapsto 0$)

$$V^{(n)} = -4\varkappa\left(\frac{\pi}{\varkappa}\right)^{2m}(2^{2m}-2) B_{2m}\delta_{n,2m}, \quad B_n = B_n(0) \tag{3.12}$$

Substitution of (3.12) into (3.11) gives a relation which is the same, apart from differences in notation, as relation (B.6) of /3/.

The Fourier transform of the soliton of the Benjamin-Ono equation $V(k) = 4\pi\exp(-|k|/c)$ does not depend analytically on the wave number, but after its expansion into terms $V(k) = 4\pi\operatorname{ch}(k/c) + 4\pi\operatorname{sgn}k\operatorname{sh}(-k/c)$, we can again obtain here an expansion (3.11) with $V^{(2m)}$ replaced by $4\pi c^{-2m}$ with supplementary terms written symbolically in the form

$$i\left(\frac{4\pi}{\tau}\right)^{1/2} \sin\left(\frac{1}{c}\frac{\partial}{\partial\eta}\right)\left[\exp\left(-\frac{\eta^2}{4\tau}\right)\operatorname{erf}\left(\frac{i\eta}{2\tau^{1/2}}\right)\right]$$

The same result can be obtained from (1.14).

In view of the complete analogy of representations (3.5), (3.6) for the mean velocity and the moments $\langle v^n\rangle$, similar expansions can be written for the latter. Assuming that the Fourier images $V^n(\eta)$ behave analytically with small wave numbers, and confining ourselves to the principal terms of the asymptotic form, we have at long times

$$\langle u\rangle \approx C_1 g(\eta, \tau),\ \langle v^2\rangle \approx C_2 g(\eta, \tau), \ldots$$
$$C_1 = \int d\eta V(\eta),\ C_2 = \int d\eta V^2(\eta), \ldots$$

For the soliton of internal waves (1.11),

$$C_1 = 4\varkappa h(1 + 1/h),\ C_2 = 8\varkappa(1 + 1/h)^2(1 - \varkappa h \operatorname{ctg} \varkappa h)$$

Using the relations along with (3.10), we can assert that, to a first approximation, we have the estimate for the velocity variance (which also remains valid for the soliton of the Benjamin-Ono equation)

$$\langle u'^2\rangle \approx \langle u_0^2\rangle + \langle v^2\rangle,\ t \to \infty$$

which shows that, at long times, the velocity pulsations are somewhat greater in the domain occupied by the soliton.

Thus, in the case of Brownian motion of the soliton, it undergoes diffusion smearing and can increase the pulsation motions of the surrounding fluid. At earlier stages, however, the effect of the presence of the soliton on the random disturbances is more considerable and more complex. For instance, immediately after switching on the random force, the disturbances increase more rapidly in front of the travelling soliton and more slowly behind it.

REFERENCES

1. WITHAM J., Linear and Non-linear Waves /Russian translation/, Mir, Moscow, 1977.
2. ABLOVITS M. and SIGUR KH., Solitons and the Inverse Problem Method /Russian translation/, Mir, Moscow, 1987.
3. WADATI M., Stochastic Korteweg - de Vries equation, J. Phys. Soc. Japan, 52, 8, 1983.
4. WADATI M. and AKUTSU Y., Stochastic Korteweg - de Vries equation with and without damping, J. Phys. Soc. Japan, 53, 10, 1984.

Translated by D.E.B.

PMM U.S.S.R.,Vol.52,No.2,pp.170-175,1988
Printed in Great Britain
0021-8928/88 $10.00+0.00

THE PHENOMENA OF TURBULENT TRANSPORT AND THE RENORMALIZATION-GROUP METHOD*

E.V. TEODOROVICH

The renormalization-group (RG) method is used to study the transport of a scalar passive impurity by turbulent velocity pulses. A solution is obtained for the turbulent Prandtl numbers, which, in the case of large-scale long-term processes (the infrared limit) tends to a universal constant, which depends only on the dimensionality of the space. The version of the RG method employed enables the behaviour of the diffusion coefficient and of the Prandtl number to be found on approaching the asymptotic mode, and for it to be shown that asymptotic RG methods can be used to describe the development of turbulence in the inertial interval of the spectrum (IIS) of wave numbers.

The ideas of the RG method made their first appearance in quantum field theory /1, 2/, and have been widely used in other fields of physics. The achievements of the method are specially clear in the theory of critical effects, the laws of which are determined by the large-scale and long-term fluctuations of the order parameter. In accordance with this, the RG technique has been developed as an asymptotic approach in which the ideas about the fixed points of the RG transformation are used and the scale similitude exponents (critical indices) are found by studying the RG transformation operator, linearized near to the fixed points /3, 4/. A similar procedure has been stated, both in the context of Wilson's approach with

*Prikl.Matem.Mekhan.,52,2,218-224,1988